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The propagation of capillary-gravity waves on a clean water surface

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Measurements are reported on the wavelength of small-amplitude water waves in a parallel-sided channel, covering the frequency range 2 to 10 Hz. Clean-surface techniques were employed in the experiments, and the results show good agreement with the predictions of linearized hydrodynamic theory.

1. Introduction

The theory of the propagation of small-amplitude long-crested waves on deep water was worked out during the last century, and represents one of the cornerstones of classical hydrodynamics. Although the simple experiments of Rayleigh and others soon gave a broad indication that the theory was correct in its predictions of wave velocity, detailed experimental investigation over a wide range of wavelengths did not follow until much later.

It was not until 1955 that Dobroklonskii reported a careful series of corroborative measurements, on clean water, covering the range 12 to 200 Hz. This large time delay in the provision of what would seem to be quite simple experimental results is mainly because of the difficulties experienced in achieving a water surface that is truly free from an elastic surface film. Indeed, even Dobroklonskii's work, using a surface continuously being renewed, is open to some criticism. However, Davies & Vose (1965) have since provided extremely well-founded experimental evidence in support of the theory up to a frequency of 900 Hz. Experimental work has recently extended the frequency of these water-wave investigations from hundreds of hertz to tens of kilohertz (Stone & Rice 1977; Byrne & Earnshaw 1980), and, although certain discrepancies have been noted between various theoretical predictions of the damping rate at these high frequencies, the wave velocity predictions are apparently acceptable.

The present work extends the observed frequency range in the opposite direction, into the capillary–gravity wave region. This extension has proved difficult, as problems are caused both by the fact that the attenuation of the waves becomes rapidly smaller as the frequency is reduced below 12 Hz and also by the increase in wavelength involved. At frequencies greater than 12 Hz the waves are quite heavily damped in progressing some tens of centimetres on a clean water surface, making wave reflections from the containing vessel unimportant. Also the relatively short wavelengths of the ripples makes it possible to use a small depth of water without seriously affecting the wave speed.

For the examination of lower frequencies, a similar measurement apparatus would need to be much larger in scale, involving the use of a much larger volume of water. This larger volume would be correspondingly more difficult to clear of surface-active materials and also much more difficult to maintain in a clean condition. Practical considerations limit the scale of clean-surface experiments to a size needing water volumes less than 10 litres (Scott 1979).

This paper reports measurements of the propagation velocity of waves in a channel of width 95 mm, using clean water of depth 29 mm. The results show good agreement with the predictions of phase velocity given by classical hydrodynamic theory, down to a wave frequency of 2 Hz.

2. The experiments

2.1. *Apparatus design*

It is not possible, bearing in mind the serious constraint on the volume of water available, to design an experiment for the wavelengths involved (up to 250 mm) without the sides of the apparatus confining the waves in some way. It was therefore decided to propagate the waves in a parallel-sided channel, with a wave-absorbing beach at one end. The choice of material for the channel was largely dictated by the need to keep the enclosed surface free from contamination. Perspex is adequate from this point of view (Scott 1979), as well as being highly convenient for precise construction. A major problem is encountered with Perspex (and in fact with any surface-clean hydrophobic material) in that the phenomenon of contact-angle hysteresis (the liquid/solid contact angle depending on the direction of motion of the liquid over the solid) modifies the wave propagation near the channel walls, tending to make the waves travel faster (Benjamin & Scott 1979). The effect of different wall materials on the propagation in channels are of considerable interest in themselves, and will be considered in a later paper. In order to avoid such effects in the present experiments, the measurements were made with perfectly wetted ground-glass plates (thickness 1 mm) held closely against the Perspex walls by the capillary pressure of the water. The glass plates were cleaned separately in strong chromic acid and rinsed with copious quantities of clean water, and it was found that the meniscus formed on them by the water in the channel always maintained a visible wet patch some 10–20 mm above the water level. The static contact angle could therefore be assumed to be zero.

The working section of the channel was 95 mm wide, 1 m long, and rectangular. The depth of water used in the experiments, 29 mm, was determined by the need to keep water in contact with the upper end of the beach with as small a meniscus as possible, to reduce reflections. It was found that, if the water was made just to touch a vertical containing wall at the top of the beach, the water layer was unlikely to break away to form a meniscus further down the beach. The beach itself was designed as an integral part of the channel, and it consisted of a linearly sloping section of length 0·5 m and slope about 2 % leading from the water contact line down to a curved step, about 20 mm high and 50 mm long, which reached the channel floor. This design proved to be an effective absorber for most of the frequency range examined in this work.

The waves were generated by a closely fitting Perspex paddle, hanging vertically from a horizontal pivot and oscillated along the channel by a small electrodynamic vibrator driven by a variable oscillator and a d.c. power amplifier. The wave period was continuously measured by an accurate counter/timer. The waves were monitored using a Wayne–Kerr capacitance probe system, a non-contact technique which essentially measured the parallel-plate capacitance between a shielded flat metal electrode and the approximately horizontal wave-bearing water surface. The Wayne–Kerr bridge (type TE200) gives an analogue output representing the water surface position. The probe electrode used in this work was specially designed to have a long thin active area ($2{\cdot}0 \times 6{\cdot}8\ \mathrm{mm}^2$) running perpendicularly across the channel. A narrow probe was chosen to give good resolution of the shorter wavelengths and to reduce the effect of surface curvature, which was thus not expected to present a problem for the wavelengths and wave amplitudes used here.

The capacitance probe was mounted on a wheeled gantry running at a constant height along a rigid frame straddling the channel, and with an earthed aluminium plate placed below the channel, stable linear surface position data was given by the system despite the low conductivity of the water used. The linear working range of of the transducer was $\pm 0{\cdot}5$ mm, and the waves observed in the present work had amplitudes less than $\pm 0{\cdot}3$ mm.

Wavelength measurements were made by examining the probe output at different positions along the channel and comparing the phase of the wave-frequency output with a fixed phase (from the source oscillator) using Lissajous figures on an oscilloscope. This phase comparison method allowed the location of successive 'zero-phase' points along the channel with a precision limited more by the scale-and-pointer arrangement used than by the observation of the Lissajous' figures. A precision of ± 2 mm is estimated for measurements of the length of wave-train observed and, as this length was always greater than $0{\cdot}5$ m, the precision of the wavelength measurements is estimated as $\pm 0{\cdot}4\,\%$. Oscillator stability was better than $0{\cdot}1\,\%$.

2.2. *Water preparation*

The water used in these experiments was distilled twice: firstly from a 3 kW Pyrex still running continuously on mains tap water, to remove dissolved inorganic materials; and secondly from a 10 litre batch apparatus containing alkaline potassium permanganate solution, to effect the removal of organic materials. The first litre of each batch was rejected as suspect, and the water was stored in chromic-acid-cleaned Pyrex glassware. All of the water used passed the sensitive 'shaking test' (see Scott 1979) for surface contamination, surface bubbles produced by shaking being observed to rupture in less than $\frac{1}{2}$ s (Kitchener & Cooper 1959).

The Perspex channel was carefully cleaned (before insertion of the ground-glass plates) using washing-up detergent on a soft sponge, followed by thorough rinsing with hot tap water and then cold clean water. As the water drained from the polished surface it drained completely to leave a visibly dry surface. This technique has been found by experience to give apparatus sufficiently free from surface-active materials for wave-damping measurements to be very reliable.

In setting up the experiment water was added to a depth greater than that required, and the excess was then removed by suction with a clean glass capillary attached to a jet pump. This suction was done both before and after insertion of the ground-glass plates (using clean disposable polythene gloves), and it gave a further opportunity for the removal of incidental surface-active contamination.

Periods (ms)	Measured Wavelength (mm)	Calculated wavelength (mm)
100	24·22	23·75
110	27·00	26·69
120	30·35	29·88
140	37·60	37·12
160	46·13	45·59
180	55·75	55·29
200	66·78	66·61
220	77·75	77·79
240	90·38	90·00
260	101·86	102·47
280	115·2	115·02
300	126·6	127·52
320	140·2	139·93
340	152·8	152·21
360	163·75	164·37
380	174·5	176·40
400	185·67	188·32
420	197·67	200·13
440	211·33	211·85
460	223·67	223·49
480	235·67	235·05
500	246·3	246·54

TABLE 1. Experimental results and theoretical predictions of the wavelength of capillary-gravity waves on clean water in a parallel-sided channel, depth 29 mm, length 1 m.

The surface tension was measured before the experiment, and at several times during an experimental run, using the apparatus described by Scott (1975). The water temperature was measured with an accurately calibrated thermometer cleaned in chromic scid.

3. The results

Wavelengths were measured for waves of periods between 100 ms and 500 ms, corresponding to the frequency range 10–2 Hz, and the results are given in table 1. Also given here are the wavelengths predicted from the first-order dispersion relation

$$\omega^2 = (gk + \sigma k^3/\rho)\tanh kh,$$

where ω is the radial frequency, k is the wavenumber (2π/wavelength), g is the acceleration due to gravity, σ is the surface tension, h is the water depth, and ρ is the water density. In the experiments, h was 29 mm and σ was 73·0 mN m^{-1} (at a temperature of 18·3 °C). Good agreement is shown, with a mean deviation of 0·75 % overall between the measured and predicted values. The agreement of theory and experiment is better for the longer-period waves (220–500 ms) than for the shorter periods.

The seven measurements in the range 100–200 ms have a mean deviation from the predicted values of 1·3 %, the waves travelling consistently faster than expected, and there are two possible explanations for this observed increase in phase velocity. Although the usual effect of surface-active contamination is to reduce wave velocities, it is possible for very low values of the surface dilatational elasticity to give a velocity increase (Lucassen-Reynders & Lucassen 1969). It is felt, however, that the precautions taken to avoid such contamination make this explanation unlikely, although it is admitted that very low levels of contamination are difficult to detect. The second possibility is that secondary flows in the channel, the result of radiation stresses, were sufficiently great to give an appreciable enhancement to the observed phase velocity. This effect would be expected to be most pronounced at the higher wave frequencies, where the phase velocities are approaching the minimum deep-water value, around 230 mm s^{-1}. Drift velocities of a few millimetres per second would be enough to give the observed discrepancies.

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